

Day 2 - PM

Integration by Parts

$$\int f'(x) g(x) dx = f(x) g(x) - \int f(x) g'(x) dx$$

Ex: Evaluate $\int x^2 \cos x dx$

$$\text{Let } f = x^2 \quad g' = \cos x$$

$$f' = 2x \quad g = \sin x$$

$$\begin{aligned} \int x^2 \cos x dx &= x^2 \sin x - \int 2x \sin x dx \\ &= x^2 \sin x - 2 \int x \sin x dx \end{aligned}$$

→ Integration by parts again!

$$f = x \quad g' = \sin x$$

$$f' = 1 \quad g = -\cos x$$

$$= x^2 \sin x - 2 \left(-x \cos x - \int -\cos x dx \right)$$

$$= x^2 \sin x - 2 \left(-x \cos x + \sin x + C \right)$$

$$= \boxed{x^2 \sin x + 2x \cos x - 2 \sin x + C}$$

Partial Fractions

$$\frac{P(x)}{Q(x)} = \frac{P(x)}{Q(x)}$$

$$Q(x) = (x-a_1)(x-a_2) \dots (x-a_n)$$

$$= \frac{A_1}{(x-a_1)} + \frac{A_2}{(x-a_2)} + \dots + \frac{A_n}{(x-a_n)}$$

$$\frac{3x-9}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

$$\text{So } 3x-9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1) \rightarrow$$

$$\text{if } x=1, \text{ then } -6 = 9A$$

$$-\frac{2}{3} = A$$

$$\text{if } x=-2 \text{ then } -15 = -3C$$

$$5 = C$$

Still need B! plug in a different value $x=2$

$$-3 = 16A + 4B + C$$

$$-3 = 16\left(-\frac{2}{3}\right) + 4B + 5$$

$$\frac{-3 - 5 + \frac{32}{3}}{4} = \frac{2}{3} = B$$

$$\text{So } \frac{3x-9}{(x-1)(x-2)^2} = \frac{-\frac{2}{3}}{(x-1)} + \frac{\frac{2}{3}}{(x+2)} + \frac{5}{(x+2)^2}$$

* try homework 2!